The Quantum World

A Rapid Introduction: Day 4

Reuben R. W. Wang [1]

[1] *Engineering Product Development*
Singapore University of Technology and Design

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Recap

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- **Quantum Circuits**: Construct and utilize quantum circuit diagrams to build quantum algorithms/protocols.
- **Q-Teleportation**: Protocol to send an arbitrary quantum state over classical channels.

Overview

1. Deutsch–Jozsa's Algorithm

2. Bell vs EPR

Deutsch–Jozsa's Algorithm: The Problem

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  Given an *oracle* function f on n bits, we want to determine if f is either
  - Constant: $f(x) = 0$ (or 1) for all values of x.
  - Balanced: $f(x) = \begin{cases} 0, & \text{half of the } x \text{ inputs} \\ 1, & \text{other half of the } x \text{ inputs} \end{cases}$

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- This is known as a **promise problem**.

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- This algorithm, would of course be the Deutsch–Jozsa's algorithm.

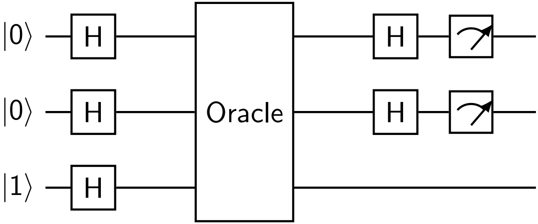
Deutsch–Jozsa's Algorithm: The Algorithm

Deutsch–Jozsa's Algorithm:

- We begin with $n+1$ qubit registers, the top n of which start in state $|0\rangle$ while the last is in state $|1\rangle$.
- We apply Hadamard gates to every qubit in the circuit.
- The oracle is now applied to all qubits as in figure.
- We again apply Hadamard gates, but now only to the top n registers.
- We measure the top n registers in the canonical basis.

Deutsch–Jozsa's Algorithm: Algorithm Analysis

Consider the 2 qubit Deutsch-Josza Algorithm:



2 Qubit Deutsch–Jozsa Algorithm

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1 Deutsch–Jozsa's Algorithm

2 Bell vs EPR

Bell vs EPR: General Spin Measurements

- We are about to discuss the proof of falsity by John Bell to the claims of Einstein, Podolsky, and Rosen (EPR).

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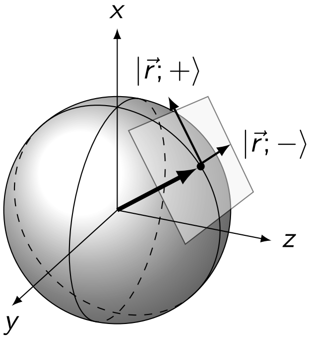

Figure: Spin Along an Arbitrary Axis

Bell vs EPR: General Spin Measurements

- It can be shown that the spin operator along this axis is given by:

$$\vec{r} \cdot \hat{\vec{S}} = \hat{S}_x \sin\theta\cos\phi + \hat{S}_y \sin\theta\sin\phi + \hat{S}_z \cos\theta = \frac{\hbar}{2}\vec{r} \cdot \hat{\vec{\sigma}}$$

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- with eigenstates:

$$|\vec{r}; +\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi} \sin\frac{\theta}{2}\,|1\rangle$$
$$|\vec{r}; -\rangle = \sin\frac{\theta}{2}\,|0\rangle - e^{i\phi} \cos\frac{\theta}{2}\,|1\rangle$$

Bell vs EPR: Activity 1

Check that $|\vec{r};+\rangle$ and $|\vec{r};-\rangle$ are indeed eigenstates with eigenvalues ± 1.

Bell vs EPR: Hidden Variables

- In 1935, Albert Einstein, Boris Podolsky and Nathan Rosen (EPR) claimed that the wave function was an incomplete description of reality.

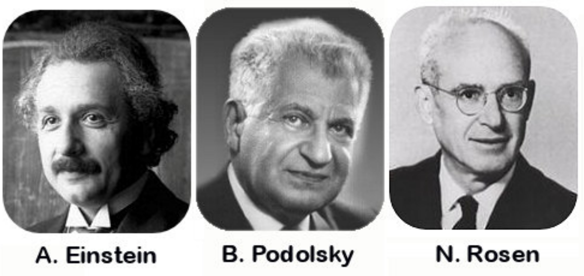

Figure: Einstein, Podolsky and Rosen

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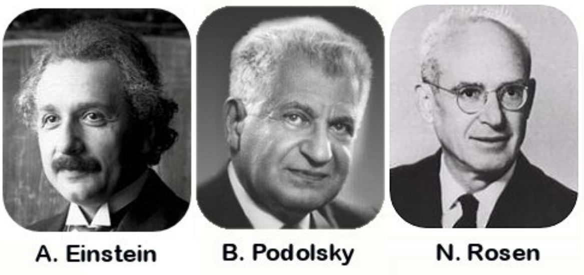

Figure: Einstein, Podolsky and Rosen

- Instead, they proposed the existence of **hidden variables** which preserved determinism (**local realism**, contradicted entanglement).

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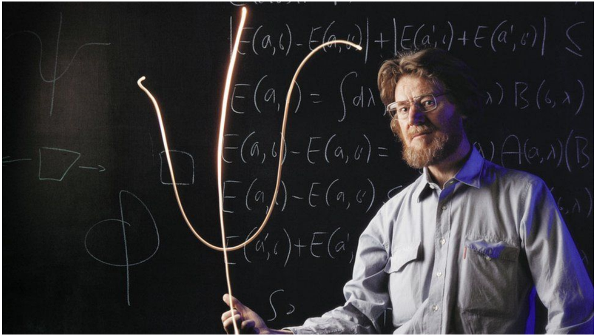
- In 1964, Bell came up with a rigorous proof that EPR's predictions could **not** be true.



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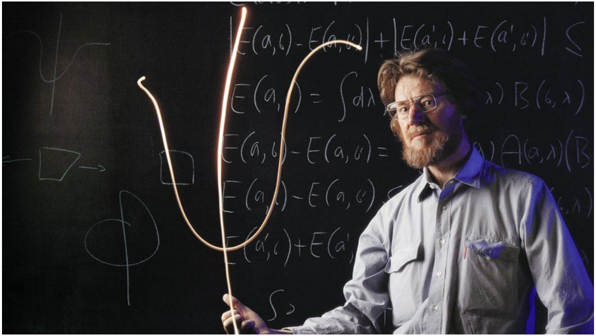

Figure: John Stewart Bell

- This was validated by the experimental works of Alain Aspect in 1981.

Bell vs EPR: Elucidation of Bell's Theorem

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- Following EPR, take a population of N particles pairs and define values $\sum_{j=1}^{8} N_j = N$.

Bell vs EPR: Elucidation of Bell's Theorem

State Population	Particle 1	Particle 2
N_1	(a, b, c)	(-a,-b,-c)
N_2	(a, b,-c)	(-a,-b, c)
N_3	(a,-b, c)	(-a, b,-c)
N_4	(a,-b,-c)	(-a, b, c)
N_5	(-a, b, c)	(a,-b,-c)
N_6	(-a, b,-c)	(a,-b, c)
N_7	(-a,-b, c)	(a, b,-c)
N_8	(-a,-b,-c)	(a, b, c)

Table: The populations of particle pairs in each unique state.

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- The equation above is known as **Bell's inequality**.

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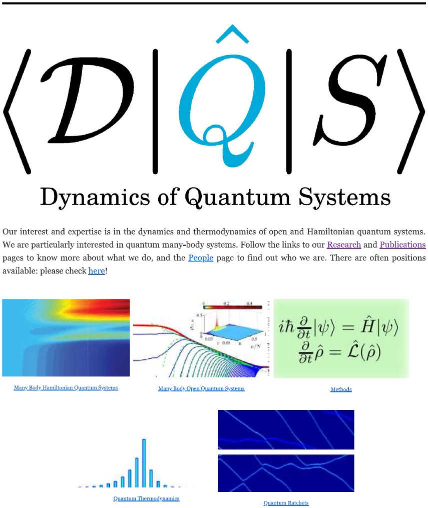
- We see that the above inequality **fails**, disproving the hidden variable theorem.

The Quantum World: Research



$\langle D|\hat{Q}|S\rangle$

Dynamics of Quantum Systems

Our interest and expertise is in the dynamics and thermodynamics of open and Hamiltonian quantum systems. We are particularly interested in quantum many-body systems. Follow the links to our Research and Publications pages to know more about what we do, and the People page to find out who we are. There are often positions available: please check here!

Many Body Hamiltonian Quantum Systems

Many Body Open Quantum Systems

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$$
$$\frac{\partial}{\partial t}\hat{\rho} = \hat{\mathcal{L}}(\hat{\rho})$$

Methods

Quantum Thermodynamics

Quantum Ratchets

Figure: Poletti Group

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Rapid Communications

Period doubling in period-one steady states

Reuben R. W. Wang,[1] Bo Xing,[1] Gabriel G. Carlo,[2] and Dario Poletti[1]

[1]*Engineering Product Development Pillar, Singapore University of Technology and Design, 8 Somapah Road, Singapore 487372, Singapore*
[2]*Departamento de Física, CNEA, Libertador 8250, (C1429BNP) Buenos Aires, Argentina*

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Nonlinear classical dissipative systems present a rich phenomenology in their "route to chaos," including period doubling, i.e., the system evolves with a period which is twice that of the driving. However, typically the attractor of a periodically driven quantum open system evolves with a period which exactly matches that of the driving. Here, we analyze a periodically driven many-body open quantum system whose classical correspondent presents period doubling. We show that by studying the dynamical correlations, it is possible to show the occurrence of period doubling in the quantum (period-one) steady state. We also discuss that such systems are natural candidates for clean and intrinsically robust Floquet time crystals.

DOI: 10.1103/PhysRevE.97.020202

I. INTRODUCTION

Classical driven and dissipative systems present a varied typology of dynamical behaviors. In these systems it is possible to observe very different types of attractors: fixed points, limit cycles, and chaotic attractors. For quantum systems, if in some limit they can be reliably described by classical equations of motion, it is also possible to observe signatures of these behaviors (see, for example, Ref. [1]).

Important types of driven dissipative systems are those for which the driving is time periodic. The steady state of such systems, when unique, has a periodicity which is given exactly by the period of the driving, even if the classical corresponding system presents period doubling or is chaotic [2]. Hence these systems deserve further investigations.

An important insight into quantum systems is given by two-time correlations. For instance, current-current correlations on a thermal state can be used to infer its linear response transport properties. For the case of quantum steady states, it was shown that the two-time correlations of a dissipative engineered quantum state can be significantly different from those of the target state [3].

Here, we show that by analyzing the two-time correlations of periodic steady states, with a period exactly given by the

a natural example of a clean Floquet time crystal, and finally in Sec. VII we draw our conclusions.

II. MODEL, PERIODIC STEADY STATE, AND MEAN-FIELD EQUATIONS

We consider a double-well potential with N atoms which is periodically driven and under the influence of dissipation. The system is described by a master equation whose time-dependent generator $\mathcal{L}_t$, of Lindblad form [10–13], is composed of two parts,

$$\dot{\hat{\rho}} = \mathcal{L}_t(\hat{\rho}) = -i[\hat{H}(t),\hat{\rho}] + \mathcal{D}(\hat{\rho}). \quad (1)$$

Note that we have set $\hbar = 1$. The first part of Eq. (1) describes the Hamiltonian evolution of the system's density operator $\hat{\rho}$, due to the Hamiltonian $\hat{H}(t)$. We consider a double well whose Hamiltonian is

$$\hat{H}(t) = -J(\hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1) + \frac{U}{2}\sum_{j=1,2} \hat{n}_j(\hat{n}_j - 1) + \varepsilon(t)(\hat{n}_2 - \hat{n}_1), \quad (2)$$

where $\hat{b}_j$ ($\hat{b}_j^\dagger$) annihilates (creates) a boson at site j, while $\hat{n}_j = \hat{b}_j^\dagger \hat{b}_j$. The Hamiltonian parameters are J, the tunneling

Figure: Period Doubling in Period-1 Steady States

The Quantum World: Research

Prospective research areas for interested parties:

- Quantum thermodynamics.
- Quantum many-body systems dynamics.
- Neural networks for quantum many-body systems.

Thank you!